GEOPHYSICS BY NEUTRINOS*)

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A review of the possibilities for the chemical exploration of the central regions of the Earth is given, making use of the antineutrino flux produced by natural radioactive isotopes.

1. INTRODUCTION

It was suggested many years ago that the neutrino and antineutrino luminosity of different celestial bodies might provide means of exploring the internal structure of these objects (see e.g. [1]). Due to the enormous mean free path the neutrinos and antineutrinos provide valuable direct information, which is not available with other methods. Searching the Sun with a neutrino telescope is well under way [2]. The present paper is concentrated on the second important task of neutrino physics: the Earth. The idea of observing terrestrial antineutrinos is not a new one [1, 3, 4, 5]. Here a review of different experimental possibilities will be given.

2. THE ANTINEUTRION SPECTRUM OF THE EARTH

The neutrino luminosity of the Earth is mainly due to electron captures in the nuclides ^{40}K and ^{138}La. This has a line spectrum, but it fades under the more brilliant neutrino rays of the Sun.

All the other β active elements show a neutron excess and are antineutrino emitters. The antineutrino spectrum of the Earth and its total antineutrino luminosity depend evidently on the chemical composition of the different strata. The relevant data are summarized in Table I. It can be seen from this table that every gram of continental crust produces one antineutrino per second. By constructing the energy spectrum of the antineutrinos from these data, one can see the following characteristics:

a) The ^{238}U and ^{232}Th families give a broad energy distribution between 0·01 MeV and 2 MeV. This is the main component of the terrestrial spectrum. The ^{235}U family produces a much lower intensity.

b) The ^{40}K decays give a rather sharp maximum near 1 MeV.

c) The ^{87}Rb decays give a maximum at about 0·2 MeV.

The contributions of the U-Th families, of the ^{87}Rb atoms and ^{40}K atoms depend evidently on the chemical concentrations of these elements. The observation of the antineutrino intensities at different eneriges enables us to calculate the overall abundancies of these isotopes inside the Earth. Even by observing the spectrum at one point, let us say, at the high energy end (above 1·3 MeV), one can calculate the total

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*) Dedicated to Professor V. Votruba on his sixtieth birthday.

Table I

Natural Beta Active Elements

Nuclide	Half-life [sec]	Ratio [%]	Max E_v [MeV]	Crust concentration [g/tons]	Specific crust activity [$\tilde{v}$/g sec]
^{238}U	$1\cdot42 \times 10^{17}$	100	α Decay	2·5	—
^{234}Th	$2\cdot07 \times 10^{6}$	66 34	0·193 0·103		
^{234}Pa	$4\cdot01 \times 10^{1}$	90 10	2·305 1·500		
^{214}Pb	$1\cdot61 \times 10^{3}$	44 56	0·659 0·590	EQUILIBRIUM	0·3
^{214}Bi	$1\cdot18 \times 10^{3}$	23 77	3·170 1·650		
^{210}Tl	$7\cdot92 \times 10^{1}$	0·04	1·960		
^{210}Pb	$7\cdot04 \times 10^{8}$	80	0·023		
^{210}Bi	$4\cdot23 \times 10^{5}$	100	1·170		
^{232}Th	$4\cdot39 \times 10^{17}$	100	α Decay	13	—
^{228}Ra	$2\cdot11 \times 10^{8}$	100	0·055		
^{228}Ac	$2\cdot23 \times 10^{4}$	12 12 35	2·100 1·760 1·180		
^{212}Pb	$3\cdot82 \times 10^{4}$	22 74 4	0·569 0·330 0·154	EQUILIBRIUM	0·18
^{212}Bi	$3\cdot65 \times 10^{3}$	54 5 1 2	2·250 1·520 0·740 0·630		
^{208}Tl	$1\cdot92 \times 10^{2}$	18 8 9 1	1·803 1·526 1·293 1·040		

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Table I continued

Nuclide	Half-life [sec]	Ratio [%]	Max E_v [MeV]	Crust concentration [g/tons]	Specific crust activity [$\tilde{v}$/g sec]
^{235}U	$2\cdot24 \times 10^{16}$	100	α Decay	0·017	—
^{231}Th	$9\cdot18 \times 10^{4}$	44 45	0·302 0·094		
^{227}Ac	$6\cdot81 \times 10^{8}$	99	0·045		
^{223}Fc	$1\cdot26 \times 10^{3}$	1	1·150	EQUILIBRIUM	0·001
^{211}Pb	$2\cdot16 \times 10^{3}$	91 2 7	1·380 0·913 0·510		
^{211}Bi	$3\cdot70 \times 10^{3}$	0·3	0·620		
^{207}Te	$2\cdot86 \times 10^{2}$	100	1·140		
^{40}K	$4\cdot01 \times 10^{16}$	89 11	1·321 K capture	3·08	0·71
^{87}Rb	$1\cdot57 \times 10^{18}$	100	0·274	148	0·46
^{138}La	$3\cdot47 \times 10^{18}$	30 70	0·205 K capture	49	0·013
^{176}Lu	$6\cdot63 \times 10^{17}$	100	0·43	0·83	0·003
					1·67

amount of heavy radioactive elements in the Earth. By comparing this result with the observed concentration in the continental crust we may find the geochemical inhomogeneity of our planet.

The integrated intensity does not appear to be very unfavourable: it amounts to about $10^9 \tilde{v}$ cm^{-2} sec^{-1} in the case of a homogeneous Earth, and may be less by a factor of 100 for a surface enhancement of the radioactivity. The 10^9 value is only one order of magnitude smaller than the solar neutrino intensity. The main difficulty is caused by the low energies of the terrestrial antineutrinos. (The radioactive elements with high decay energies have short life-times and have disappeared in the many billion years of terrestrial history.) The detection possibilities of antineutrinos in the KeV — MeV region should be studied carefully.

3. INDUCED BETA DECAY

The classic method of neutrino and antineutrino detection is the induced β decay [6]:

(1) $$\tilde{v} + X \to Y + e^+ .$$

If the target nucleus is stable against electron capture, i.e. if the reaction

$$e^- + X \to Y + v$$
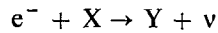
is energetically forbidden, the energy threshold of the reaction (1) must be definitely larger than $2m_e c^2 = 1 \cdot 022$ MeV. In this case the decay

(2) $$Y \to X + e^- + \tilde{v}$$

is energetically allowed. The appearance of the positron and of the radioactive nucleus Y in the final state makes transition (1) observable. The observed life-time of the inverse reaction (2) informs us also about the value of the nuclear matrix element in the weak transition (1).

From the theory of weak interactions the cross-section of the reaction (1) is given by

(3) $$\sigma(\varepsilon) = \frac{2\pi^2 \lambda_c^3 \ln 2}{ft c} (\varepsilon - \varepsilon_0) \sqrt{[(\varepsilon - \varepsilon_0)^2 - 1]} \quad \text{(for } \varepsilon > \varepsilon_0 + 1) .$$

Here

$$\frac{E_0}{m_e c^2} = \varepsilon_0 = \frac{M_X - M_Y}{m_e} ,$$

t is the half-life and ft is the reduced half-life of the nucleus Y against the spontaneous decay (2), c is the speed of light and $\lambda_c = \hbar/m_e c$ is the Compton wave length. (An allowed transition is considered, the Coulomb wave distortion of the positron and electron wave functions is neglected.) If $I(\varepsilon)$ dε is the spectral distribution of the antineutrino current density (in $\tilde{v}$ cm^{-2} sec^{-1} units), the transition probability of the target nucleus (in sec^{-1} units) is given by the formula

(4) $$T^{-1} = \int_{\varepsilon_0 + 1}^{\infty} \sigma(\varepsilon) I(\varepsilon) \, d\varepsilon =$$

$$= \left[4\pi^3 \lambda_c^3 \frac{\ln 2}{ft c} \right] \frac{1}{2\pi} \int_{\varepsilon_0 + 1}^{\infty} (\varepsilon - \varepsilon_0) \sqrt{[(\varepsilon - \varepsilon_0)^2 - 1]} I(\varepsilon) \, d\varepsilon .$$

It is shown by this formula that the detector is sensitive only for neutrinos with energy $E_{min} > (\varepsilon_0 + 1) \, m_e c^2$. In principle, the most convenient target is the ^{3}He nucleus [4]:

(5) $$\tilde{v} + {}^3\text{He} \to {}^3\text{H} + e^+ \quad (E_{min} = 1 \cdot 041 \text{ MeV}, \log ft = 3 \cdot 03)$$
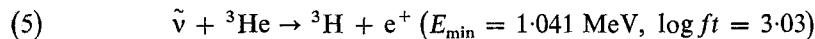
(a superallowed transition with a threshold as low as possible). For practical use the allowed low energy reaction

$$\text{(6)} \qquad \tilde{v} + {}^{35}\text{Cl} \rightarrow {}^{35}\text{S} + e^{+} \quad (E_{min} = 1\cdot189 \text{ MeV}, \ \log ft = 4\cdot98)$$
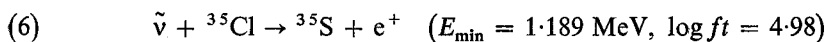
or the superallowed medium energy reaction [3, 4, 6]

$$\text{(7)} \qquad \tilde{v} + {}^{1}\text{H} \rightarrow n + e^{+} \quad (E_{min} = 1\cdot804 \text{ MeV}, \ \log ft = 3\cdot13)$$
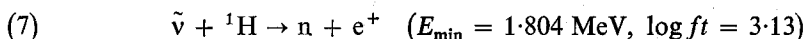
may be considered. These examples show that only the high energy end of the terrestrial antineutrino spectrum is accessible with the induced β decay technique. Most antineutrinos are lost. This fact makes the value T^{-1} rather small in spite of the fact that the total terrestial intensity

$$\text{(8)} \qquad I_0 = \int_0^{\infty} I(\varepsilon) \, d\varepsilon$$

is quite considerable. For observing the U-Th antineutrinos purely $(E_v > 1\cdot3 \text{ MeV}$ region) the classical target ${}^{1}\text{H}$ is convenient. The isotopes ${}^{234}\text{Pa}$ and ${}^{214}\text{Bi}$ from the U family give the largest contribution.

4. INDUCED ELECTRON CAPTURE

Enrico Fermi mentioned already in 1949 in this famous Chicago lectures [5, 7] that an incoming antineutrino could induce a K capture on an otherwise stable nucleus X:

$$\text{(9)} \qquad \tilde{v} + e^{-} + X \rightarrow Y \, .$$

Here the electron from the K shell helps in raising the nucleus from the energy state M_X into the energy state M_Y, so the energy of the captured antineutrino is uniquely given by the formula

$$\text{(10)} \qquad E_v = E_0 - m_e c^2 + E_K \, .$$

(E_0 is defined by Eq. (3), E_K is the binding energy of the electron in the K shell. For a smooth $I(\varepsilon)$ the term E_K may be neglected.) The energy (10) lies lower by 1 MeV than the threshold of the induced β decay (1). Thus the reaction (8) is convenient for counting the soft antineutrinos.

The transition probability per unit time between two definite quantum states in the reaction (9) is given by the formula

$$\text{(11)} \qquad w(E) = \frac{4\pi^4}{V} \frac{\ln 2}{ft} \lambda_c^6 m_e c^2 |\psi_e(0)|^2 \ \delta(E + m_e c^2 - E_0) \, .$$

Here ft is again the reduced half life-time of the inverse decay $(2,)$ $\psi_e(x)$ is the electron wave function in the K shell and V is the normalization volume. If $n(E)$ is the (averaged) occupation number of the antineutrino states (being $0 \leq n(E) \leq 1$ from the Pauli principle), the mean life-time T of the nucleus X swimming in the antineutrino sea characterized by $n(E)$ may be written as follows:

$$(12) \qquad T^{-1} = \int w(E)\ n(E)\ \frac{V\,\mathrm{d}^3 p}{h^3}\ .$$

The relation between the occupation number $n(E)$ and the spectral intensity $I(\varepsilon)$ is given by the equation

$$(13) \qquad I(\varepsilon)\ \mathrm{d}\varepsilon = c\ n(E)\ \frac{\mathrm{d}^3 p}{h^3} = \frac{4\pi c}{(2\pi \lambda_c)^3}\ n(E)\ \varepsilon^2\ \mathrm{d}\varepsilon\ .$$

Combining the equations (11), (12) and (13) one arrives at the final formula:

$$(14) \qquad T^{-1} = \left[4\pi^3 \lambda_c^3\ \frac{\ln 2}{ftc} \right] \left(\frac{Z}{137} \right)^3 I(\varepsilon_0 - 1)\ .$$

(Here the Schrödinger value

$$|\psi_e(0)|^2 = \frac{1}{\pi} \left(\frac{Z}{137\lambda_c} \right)^3$$

has been used. For high Z values the Dirac wave functions would give more accurate results.)

Formula (4) shows that the induced β decay technique gives integrated information on the $E_\nu > E_0 + m_e c^2$ tail of the antineutrino spectrum; the induced electron capture technique (14), on the other hand, informs us about the value of the spectral intensity at the point $E_\nu = E_0 - mc^2$. The sensitivity of the two techniques (given by the numerical factor in brackets) is practically the same. By choosing different atoms one is able to measure the spectral intensity point by point. The efficiency of the different detectors is given by their Z^3/ft values. This means that the stable daughter nuclei of the allowed β decays (leading into the nuclear ground state) at the end of the periodic system are the most convenient detectors. Some possibilities are listed as follows:

^{3}He $\rightarrow$ H	$E_\nu = 0\cdot02$ MeV	$Z^3/ft = 0\cdot01$
^{35}Cl $\rightarrow$ ^{35}S	$0\cdot17$ MeV	$0\cdot05$
^{45}Sc $\rightarrow$ ^{45}Ca	$0\cdot25$ MeV	$0\cdot01$
^{130}Ba $\rightarrow$ ^{130}Cs	$0\cdot44$ MeV	$2\cdot42$
^{209}Bi $\rightarrow$ ^{209}Pb	$0\cdot63$ MeV	$1\cdot81$
^{112}Sn $\rightarrow$ ^{112}In	$0\cdot65$ MeV	$9\cdot93$
^{164}Er $\rightarrow$ ^{164}Ho	$0\cdot97$ MeV	$1\cdot98$
^{207}Pb $\rightarrow$ ^{207}Tl	$1\cdot43$ MeV	$4\cdot38$
^{206}Pb $\rightarrow$ ^{206}Tl	$1\cdot50$ MeV	$4\cdot38$

It can be seen that the ^{130}Ba, ^{112}Sn and ^{164}Er are convenient detectors to measure the antineutrinos coming from the ^{40}K decays (max $E_\nu = 1\cdot321$ MeV, intensity maximum in the $0\cdot5-1$ MeV region). The antineutrinos of the ^{87}Rb decay may be caught only by the inefficient ^{35}Cl target (max $E_\nu = 0\cdot274$ MeV).

5. GEOCHEMICAL CONSIDERATIONS

The spectral intensity function $I(\varepsilon)$ of terrestrial antineutrinos, and consequently the actual values of the T life-times of the different target nuclei, depend on the (unknown) $\varrho(r)$ density distribution of the Earth and the (unknown) $c(r)$ mass concentrations of the radioactive elements in it. Just to obtain a general idea of the orders of magnitude, let us assume a homogeneous Earth.

Model A: $\varrho = $ const; $c = $ const and equal to the values observed in the continental crust. In this case the total terrestrial $\tilde{\nu}$ intensity at sea level amounts to

$$I_0^{(A)} = 3 \times 10^9 \; \tilde{\nu} \, \text{cm}^{-1} \, \text{sec}^{-1} \,,$$

a value comparable to the solar ν intensity reaching the Earth. As a consequence of the lower energies the T^{-1} values are considerably smaller. For orientation we quote the T^{-1} values of certain target isotopes in Table II. It can be seen that this Earth model predicts T^{-1} values lying between 10^{-34} and 10^{-37} for the most convenient targets. This prediction may be compared with the present sensitivity record, achieved by the group of R. Davis [2], working on the solar neutrino experiment:

$$T^{-1} \leqq 3 \times 10^{-36} \, \text{sec}^{-1}$$

was measured for the ν capture reaction

$$\nu + {}^{37}\text{Cl} \rightarrow {}^{37}\text{A} + e^-$$
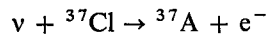
with a brilliant radiochemical technique.

Table II

Transition Probabilities of Different Target Nuclei for *Model A*

Target	T^{-1} for ^{40}K	T^{-1} for ^{87}Rb	T^{-1} for U + Th	total T^{-1}
^{35}Cl	$3\cdot7 \times 10^{-40}$	$2\cdot7 \times 10^{-40}$	$9\cdot9 \times 10^{-40}$	$1\cdot6 \times 10^{-39}$
^{209}Bi	$9\cdot3 \times 10^{-38}$	—	$1\cdot5 \times 10^{-38}$	$1\cdot1 \times 10^{-38}$
^{112}Sn	$5\cdot4 \times 10^{-37}$	—	$7\cdot8 \times 10^{-38}$	$6\cdot2 \times 10^{-37}$
^{164}Er	$1\cdot1 \times 10^{-37}$	—	$1\cdot4 \times 10^{-38}$	$1\cdot2 \times 10^{-37}$
^{207}Pb	—	—	$2\cdot2 \times 10^{-38}$	$2\cdot2 \times 10^{-38}$
^{3}He	$2\cdot3 \times 10^{-35}$	—	$4\cdot8 \times 10^{-35}$	$7\cdot1 \times 10^{-35}$
^{35}Cl	$5\cdot1 \times 10^{-38}$	—	$3\cdot6 \times 10^{-37}$	$4\cdot1 \times 10^{-37}$
^{1}H	—	—	$4\cdot2 \times 10^{-36}$	$4\cdot2 \times 10^{-36}$

Model A certainly overestimates the terrestrial $\tilde{v}$ flux. A homogeneously distributed radioactivity would produce an enormous temperature everywhere in the Earth. According to the general opinion of geochemists the radioactivity is more or less concentrated in the surface layers of the Earth. We mention three possible inhomogeneous Earth models:

Model B: The radioactive elements are completely concentrated in the surface layer of the Earth. For example, a radioactive crust 30 km thick may be assumed.

Model C: The radioactive elements are completely concentrated in the continental crust, being, let us say, 30 km thick and covering about 1/3 of the surface of the Earth.

The last-mentioned models are probably very restrictive. In a realistic model one may assume a higher concentration of the radioactive elements in the crust and a lower concentration in the inner parts of the Earth. For example, Eder tried to calculate the total amount of radioactive elements from the assumption that the observed slow expansion of the Earth is due to the heat produced by the radioactivity (*Model D*) [3].

The antineutrino intensities of the different models and the corresponding T^{-1} values may be obtained by multiplying the corresponding results of *Model A* by the factor γ which is given by the formula

$$(15) \qquad \gamma = \frac{1}{R} \int_0^R q(r) \ln \frac{R + r}{R - r} \, dr .$$

Here $q(r)$ is constructed from the actual mass density $\varrho(r)$, the mean density $\langle \varrho \rangle$, the actual concentration of the radioactive elements $c(r)$ and the surface concentration $c(R)$ as follows:

$$q(r) = \frac{c(r)}{c(R)} \frac{\varrho(r)}{\langle \varrho \rangle} .$$

R is evidently the radius of the Earth. With the help of formula (15) we can calculate the correction factors to the T^{-1} values quoted in Table II. These factors are as follows:

Model A	$\gamma = 1$
Model B	$\gamma = 0 \cdot 03 \varrho(R)/\langle \varrho \rangle$
Model C	$\gamma = 0 \cdot 01 \varrho(R)/\langle \varrho \rangle$
Model D	$\gamma = 0 \cdot 5$

Just the model dependence of the factor γ enables us to make a difference between the homogeneous and inhomogeneous Earth models experimentally.

The observable inhomogeneity of the radioactivity in the Earth offers us means of collecting valuable information about the formation of the planets. In the case of cold formation a rather uniform distribution of the chemical elements may be expected, hot formation could make a stronger separation possible. The concentration

of radioactive elements in the centre of the Earth is of decisive importance for the thermal history of our planet.

The origin of the planetary system is one of the greatest puzzles of contemporary astronomy. The antineutrino luminosity of the Earth contains direct information for the solution of this puzzle. It thus seems to be worth considering the detection of this glow of our little planet even if the capture of terrestrial antineutrinos is at the very limits of present experimental possibilities.

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